B. Sc - part - I (sub) Set Theory 09-07-2020

Partition of a set

Q.1 Define Partition of a set with an example.

Soln: Partition of a Set: →

Let S be a non-empty set.

A set $P = \{A, B, C, \ldots\}$ of non-empty subsets of S will be called a partition of S if

(I) $A \cup B \cup C \cup \ldots = S$

that is, the set S is the union of the sets in P

(II) The intersection of every pair of distinct subsets of $S \in P$ is the null set.

That is, if $A, B \in P$, then either $A = B$

or, $A \cap B = \phi$.

For example: Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and its subsets $A_1 = \{1, 2\}$, $A_2 = \{3, 4, 5\}$, $A_3 = \{6, 7, 8, 9\}$, $A_4 = \{10\}$. Then the set $P = \{A_1, A_2, A_3, A_4\}$ is such that

(I) A_1, A_2, A_3, A_4 are all non-empty subset of S,

(II) $A_1 \cup A_2 \cup A_3 \cup A_4 = S$ and

(III) either $A_i = A_j$ or, $A_i \cap A_j = \phi$.

Thus, we can say that the set $\{A_1, A_2, A_3, A_4\}$ is a partition of S.

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Equivalence Relations.

Q.2. Fundamental theorem in Equivalence Relations.

Soln: Statement:→ An equivalence relation R in a non-empty set S determines a partition of S and conversely a partition of S defines an equivalence relation in S.

Proof: Since R is an equivalence relation in a set S we can have equivalence classes S_a or $[a]$ such that

$$[a] = \{x : x \in S \text{ and } xRa\}$$

Let A be the set of equivalence classes of S with respect to R.

$$\therefore A = \{[a] : a \in S\}$$

Now, we have to prove that this A is a partition of set S for which we have to prove that

(i) all sets of A like $[a]$ are non-empty

(ii) $\bigcup [a] = S$ and

(iii) $a \in S$ sets of A are pairwise disjoints.

(i) Since R is an equivalence relation which is reflexive. We have aRa.

$\therefore \forall a \in S$, We have aRa. $[a]$ is non-empty set.

Hence, $a \in [a]$ and thus $[a]$ is non-empty set.

(ii) Every element a of S is an element of the equivalence class $[a]$ in A and hence S is the union of all ~~equivalen~~ equivalence classes i.e. $S = \bigcup\limits_{a \in S} [a]$

(iii) If $[a]$ and $[b]$ are two equivalence classes then either $[a] = [b]$ or $[a] \cap [b] = \phi$

Hence, A is a partition of set S. Thus, we see that an equivalence relation R in S decomposes the set S into equivalence classes.